The Role of Simplicity in the Historical Development of the R-2R DAC Ladder Network

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Abstract

Digital-to-analog conversion (DAC) has played a pivotal role in modern electronics, bridging the gap between binary and analog signals. The development of the R-2R DAC in particular demonstrated the advanced yet simplistic nature of DAC circuits. Previous models of DACs before the R-2R were superseded by the R-2R DAC because of its "simplistic" architecture, specifically its ability to be constructed with just two resistors, R and 2R. This basic nature allows it to be reduced to a circuit with a single resistor value of R, allowing the DAC to take in higher amounts of bits as digital input (PCM) by just adding another R and 2R, while maintaining its binary weighted nature, instead of having resistor values that become impossible to manufacture. This paper will focus specifically on the R-2R DAC ladder network and all of its historical references, offering an insight on the development of DACs.

I. Predecessors of R-2R DAC

The concept of digital-to-analog conversion existed long before the term "DAC" even entered the technical lexicon. The Kelvin-Varley divider, named after its two inventors, Baron Kelvin and Cromwell Fleetwood Varley, is an early example of a voltage divider. Used predominantly in the early 1930's, the Kelvin-Varley divider's architecture involves stages of equal amounts of resistors. The stages are connected in chains. Once the KVD receives a voltage input, it can use wipers to cut off a fraction of the voltage. For example, if the 5th resistor is cut off with the wiper, 50% of the initial voltage is kept. This process is repeated, depending on the number of resistor stages, until a highly precise voltage output is achieved. Although extremely accurate, the KVD is prone to malfunction when exposed to temperature changes. It is also hard to manufacture with its complex architecture and high amounts of resistor values.

Binary-weighted DACs were the first ever electrical functioning DAC, coming after the Kelvin-Varley divider. Although conceptually simple, the binary-weighted DAC is also difficult to manufacture and maintain due to its architecture. John C. Schelleng's 1946 patent for a 6-bit PCM system implemented an early form of vacuum tube binary DAC. The binary-weighted DAC takes a binary PCM input from MSB (most-significant bit) to LSB (least-significant bit) and puts it through its stages for a voltage analog output. Its stages vary by resistor value, starting from R and doubling every stage: R, 2R, 4R, 8R, 16R… The MSB pairs with the lowest resistor value and the LSB pairs with the highest. This gives the MSB the most current output because more current can flow through the low resistors. Much like the early Kelvin-Varley Divider, the usage of the binary-weighted DAC was impractical due to the difficulty in manufacturing. The scale of the resistors would often extend into larger numbers, making DACs of higher resolutions hard to produce. Although it provided an electric analog output in a precise manner, it still lacked simplicity.

II. Overview of the R-2R DAC

The R-2R DAC Ladder Network's robust nature allows it to function conceptually similar to the binary-weighted DAC but only requires two types of resistors for the entire network, R and 2R. There are two modes for the R-2R DAC, a current mode and a voltage mode. For the sake of simplicity, this paper

will only go over the voltage mode. Information regarding the current mode of R-2R DACs can be found in Erb & Wierzba [1].

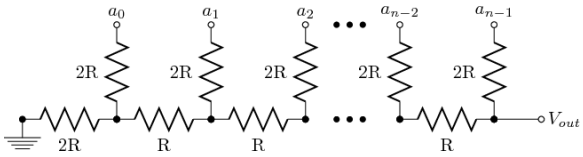


Fig. 1. R–2R resistor ladder DAC schematic [Adapted from Wikipedia, CC-BY-SA 3.0].

The voltage-mode R-2R DAC resistor ladder takes a direct binary input, often in the form of a PCM, and converts it to an analog signal. The architecture of the R-2R is set in a way that allows it to be reduced to a simple circuit with one resistor value R, after applications of Thevenin's theorem and basic source transformation. This relative architecture allows the R-2R to attach other stages of R and 2R resistor values for each additional bit, allowing it to be simpler yet remain efficient. An R-2R receives the digital input in the form of binary, such as 001 for a 3-bit R-2R. The voltage output for a 3-bit DAC with an input of 001 will be:

$$V_{ref} \times \left(\frac{B2}{2} + \frac{B1}{4} + \frac{B0}{8}\right) = V_{out}$$

$$5V \times \left(\frac{0}{2} + \frac{0}{4} + \frac{1}{8}\right) = 0.625V$$

B2 represents the MSB, and B0 represents the LSB. The binary input of 001 goes from MSB to LSB. B2 weighs more for the total voltage output, thus having the name "most significant bit".

III. Historical References of R-2R DAC

It is wrong to state that the R-2R DAC was the first one of its kind, but its simplicity and design was innovative. The main issues of its predecessors, such as the binary-weighted DAC, involved difficulties with manufacturing and construction due to costs and production faults of high resistor values, especially for high-bit DACs that reached figures upwards of 16 bits. Ultimately, the R-2R DAC solved this issue with its structure of simply R and 2R, further superseding older forms of DACs. The architecture of the R-2R was first introduced in 1953 by B. D. Smith [2]. In his paper, Smith showed how exponentially scaled resistors were unnecessary with the placement of the R and 2R values. Smith provided the earliest overview of the construction of R-2R, and its first implementation in a circuit was seen in a patent filed in 1955 and granted in 1963 by Robert P. Talimbaras and Bernard M. Gordon [3].

R-2R DACs were first implemented in military defense and instrumental systems before spreading into computing and electronics. Although solving many of the problems previous DAC types had, the R-2R DAC posed challenges of its own. Once higher-resolution DACs became more desirable, R-2R DACs soon began competing with other DAC types, such as sigma-delta DACs. The R-2R DAC struggled with resolutions above 16 bits due to its high-precision resistor-matching architecture. A minuscule mismatch could lead to errors in producing the correct voltage output. The R-2R DAC also created "glitches" in higher resolution systems. Glitches are created by the sudden switching of many bits, causing spikes that lead to inaccurate outputs. Newer DACs, the sigma-delta DAC, for example, fixed this problem by running on a simple 1-bit DAC that could be run at high oversampling rates. In higher resolutions of upwards of 20 bits, sigma-delta DACs were superior. Nowadays, R-2R DAC networks are still used in audio and mixed-signal ICs where mid-resolution systems are usable.

Compiled below are all references to the R-2R DAC Ladder Network in chronological order, focused on how each reference built upon its predecessor and its contributions. Each entry has publication details.

1953: B. D. Smith — "Coding by Feedback Methods." First ever reference of a "ladder resistor decoding network", essentially the R-2R DAC. Introduced the idea of binary scaling and laid the foundation for others.

1955 / 1963: Robert P. Talimbaras and Bernard M. Gordon, U.S. Patent 3,108,266 — "Signal Conversion Apparatus." First ever real-life implementation of the R-2R DAC, laid inside a SAR DAC. Showed the possibility of real hardware implementation.

1980: J. Zurada & K. Goodman, Electronic Letters — "Equivalent circuit of multiplying DAC using R–2R ladder networks." Shows the possibility of

multiplying DACs with R-2R with an equivalent circuit.

1983: E. Erb & G. Wierzba, IEEE Trans. Circuits Syst. — “Expression for the Output Resistance of a Switched R–2R Ladder Network.” Early introduction of the current mode of R-2R DACs. Heavy analysis of the output resistance of the current mode. Contributed greatly to the error analysis of DAC.

1985: V. V. B. Rao & K. S. Rao, IEEE Trans. Circuits Syst. — “Equivalent circuit for a multiplying D/A converter.” Further expanded the idea of multiplying DACs while discussing methods on increasing the accuracy of R-2R current-mode performance predictions.

2000: M. Kennedy, IEEE Trans. Circuits Syst. I — “On the robustness of R-2R ladder DACs.” Analyzed the robustness of the R-2R by testing it under non-ideal circumstances. Was an important “reliability” study.

2001: L. Wang, Y. Fukatsu, K. Watanabe, IEEE Trans. Instrum. Meas. — “Characterization of CMOS R-2R ladder digital-to-analog converters.” Experimental results of the current-mode of R-2R DACs proving its feasibility in circuits.

2006: D. S. Karadimas et al., Proc. ISCAS — “A digitally calibrated R-2R ladder architecture for high performance DACs.” Improved accuracy of high-performance DACs by proposing a “digitally calibrated R-2R ladder structure”.

2008: D. Marche, Y. Savaria, Y. Gagnon, IEEE Trans. Circuits Syst. I — “Laser fine-tuneable deep submicron CMOS 14 bit DAC.” Used an R-2R ladder DAC to create a deep-submicron CMOS 14-bit DAC. Pushed DAC resolution limits.

IV. Conclusion

The historical development of the R-2R DAC ladder network proved to be essential in the advancement of digital-to-analog converters. From the manufacturing difficulties of the Kelvin-Varley divider to the relatively limited architecture of the binary-weighted DAC, the R-2R DAC provided an innovative way to convert binary input into direct voltage outputs. With the same functionality as its predecessors, the R-2R remained easy to manufacture even at higher bit resolutions.

Following B. D. Smith’s “Coding by Feedback Methods” in 1953 [2] and later U.S. Patent 3,108,266 [3], the R-2R DAC began to be widely used in standard military and computing instrumentation. Further historical developments focused on how the R-2R DAC could be even more applicable, especially when higher resolutions were needed. Many tests on how the R-2R would perform in non-ideal situations, introducing a “digitally calibrated” R-2R structure, and tests on the current mode of the R-2R were all conducted to push the R-2R’s resolution limits. However, the introduction of a more compatible sigma-delta DAC and other advanced DAC types soon replaced much of the R-2R’s usage in real-life applications that required higher resolution types.

Today, the R-2R continues to remain prevalent in computations where mid-resolution types suffice. The historical development of the R-2R DAC provides insight into how simplicity often works better than complexity. The subsequent refinements added by researchers to improve it shows how evaluating trade-offs in precision, manufacturability, and functionality will continue to drive innovation in circuit engineering.

V. References

[1] E. Erb and G. Wierzba, "Expression for the output resistance of a switched R-2R ladder network," IEEE Trans. Circuits Syst., vol. CAS-30, no. 5, pp. 341-342, May 1983.

[2] B. D. Smith, "Coding by feedback methods," Proc. IRE, vol. 41, no. 10, pp. 1053-1058, Oct. 1953.

[3] B. M. Gordon and R. P. Talambiras, "Signal conversion apparatus," U.S. Patent 3,108,266, Oct. 22, 1963.